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Excited States and Electromagnetic Transition of A= 78 Calculated Using Shell Model and BCS Theory

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Abstract: **Shell model calculations with BCS theory were performed to study the first exited states and the reduced transition probabilities $B(M\lambda; J_i \rightarrow J_{gs})$ for the A= 78 chain of isobars by employing the MSDI as a residual interaction between two quasiparticles, in the pure configurations and the configuration mixing for all allowed states on the full valence space $1p\text{-}0f\text{-}0g_{9/2}$. The core is taken at $^{40}_{20}Ca$ for all nuclei under study and the results of our theoretical calculations are compared with the most recent available experimental data. It was found that the shell model using BCS theory and MSDI is still one of the most suitable models to carry out, such as calculations, and taking configuration mixing into consideration, enhances considerably, the results, more than the case of pure configurations when comparison with experimental data, and we found success for MSDI as a residual interaction even when the valance nucleons is large.**

Keywords: **configuration mixing, modified shell model, quasiparticle, transition probability, occupation amplitude.**

1. INTRODUCTION

The nuclear shell model has been very successful in our understanding of nuclear structure once a suitable effective interaction is found, the shell model can predict various observables accurately and systematically. For light nuclei, there are several standard effective interactions, such as the Cohen-Kurath and the USD interactions for p and sd shells, respectively. On the other hand, in the $1p\text{-}0f\text{-}0g_{9/2}$ space, there are, also, standard interactions, such as FPD6, GXPF1[1] and Modified Surface Delta Interaction (MSDI)[2]. Shell model calculations have been performed recently with the MSDI as an effective two-body interaction for many nuclei[2]. During the last three decades, the shell model configuration mixing calculations have yielded extremely valuable contributions to the microscopic understanding of many nuclear structure properties. However, it is well known that the configuration mixing approach is restricted to rather small valance spaces or comparable basis systems due to the very large dimensions of the matrices that need to be diagonalized. Recent technological innovations have extended the shell model calculations up to A~60 region, where the energy spectra and other properties of nuclei can be studied by exact diagonalizations in a full major oscillator shell. Because of the much larger configuration space required, the heavier nuclei cannot be studied by using this procedure yet. Even if it may technically be attainable on a modern supercomputer, such a calculation is not of much interest from a physical point of view, because it is very difficult to guarantee that the data obtained in this way are able to uncover the physics hidden behind a vast amount of computer output[3]. In order to overcome the above mentioned, many approaches have been developed and extensively applied to investigate the structure of various nuclei. Such as the BCS(Bardeen, Cooper, Schrieffer) theory[4], using this theory to treat the problems of the nuclear pairing correlation is considered to be suitable for a system containing a large number of particles. This method have achieved great success in describing the energy spectra, the electromagnetic properties, and some other important structure phenomena of nuclei. Because of the quite importance of the $1p\text{-}0f\text{-}0g_{9/2}$ space for variety of problems in nuclear structure, this space is a region where the shell model can play an indispensable role and is at the frontier of our computational abilities.

In the $1p\text{-}0f\text{-}0g_{9/2}$ space one finds the interplay of collective and single-particle properties. This region is also of special interest from the viewpoint of astrophysics, such as supernovae explosions[5].

The aim of the present work is to study the first exited states and the reduced transition probabilities $B(M\lambda; J_i \rightarrow J_{gs})$ in the A= 78 chain of isobars, which are ${}^{78}_{34}Se, {}^{78}_{35}Br$ and ${}^{78}_{36}Kr$ nuclei by employing BCS theory and the MSDI as a residual interaction between two quasiparticles, in the pure configurations and the state of configuration mixing for all allowed states on the full valence space $1p\text{-}0f\text{-}0g_{9/2}$ using the shell model code AUSM(Aleppo University Shell Model) which invited by us.

2. THE NUCLEAR SHELL MODEL

The nuclear shell model, introduced almost 50 years ago by Mayer Haxel, Jensen, and Suess, has been very successful in describing the properties of nuclei with few valence nucleons[6]. These properties include the energy levels, magnetic and quadruple moments, electromagnetic transition probabilities, beta decay, and cross section for various reactions.

The basic assumption of the nuclear shell model is that, to a first approximation each nucleon moves independently in a potential that represents the average interaction with the other nucleons in a nucleus. The complete Schrodinger equation for A nucleons reads as

$$\hat{H}\,\Psi(1,2,...,A) = E\Psi(1,2,...,A) \quad (1)$$

where $\Psi(1,2,...,A)$ is a totally antisymmetric wave function and $\hat{H}$ contains single nucleon kinetic energies and two-body interactions as

$$\hat{H} = \hat{H}^{(0)} + \hat{H}^{(1)} = \sum_{i=1}^{A} \hat{h}(i) + \hat{V} \qquad (2)$$

$\hat{H}^{(0)} \equiv \sum_{i=1}^{A} \hat{h}(i)$ denoted a sum of single-particle Hamiltonians and $\hat{H}^{(1)} \equiv \hat{V}$ is called a residual interaction.

2.1. Shell Model with Pure Configurations:

In the naive shell model we neglect the residual interaction $\hat{V}$, However, since we are dealing with the fermions of two sorts, protons and neutrons, the correct shell model wave function should be antisymmetric under permutation of any two nucleons with respect to its space, spin and isospin coordinates and it should possess definite values of the total angular momentum with parity and the total isospin $\Gamma = (J^{\pi}, T)$. So, we can construct the final shell model wave functions as certain linear combinations of functions totally antisymmetric. We will denote them as $\Phi_{\Gamma}(\vec{r}_1, \vec{r}_2, \vec{r}_3, ...\vec{r}_A)$, This antisymmetrized wave function is called the Slater determinant of the given single-particle states, which reads as

$$\Phi_{\Gamma}(\vec{r}_1, \vec{r}_2, \vec{r}_3, ..., \vec{r}_A) = \chi[\prod_{i=1}^{A} \varphi_{\alpha_i}(\vec{r}_i)] \qquad (3)$$

Here χ is an antisymmetrization operator that performs the sign accompanied permutations of the single-particle orbitals in the product wave function and also carries a normalization factor. The wave function of a nuclear state was taken to be a Slater determinant corresponding to a definite way of placing the nucleons in the mean field single-particle orbitals which called "pure configuration". In this model, The single-particle wave functions from Slater determinant are solutions of the corresponding Schrodinger equations is given as[7]

$$\hat{h}\varphi_{\alpha}(\vec{r}_i) = \varepsilon_{\alpha}\varphi_{\alpha}(\vec{r}_i) \qquad (4)$$

Here α labels the single-particle state $|n\ \ell\ s\ jm\ tm_t\rangle$ and ε_{α} is single-particle energy.

The Schrodinger equation for pure configurations reads as

$$H^{(0)}\Phi_{\Gamma}(\vec{r}_1, \vec{r}_2, \vec{r}_3, ...\vec{r}_A) = E_{\Gamma}\Phi_{\Gamma}(\vec{r}_1, \vec{r}_2, \vec{r}_3, ...\vec{r}_A) \qquad (5)$$

The total energy is thus given as

$$E_{\Gamma}^{(0)} = \sum_{i=1}^{A} \varepsilon(i) \qquad (6)$$

Such a model is useful to get simple single-particle estimations of different physical observables, assuming that the properties are determined only by the last nucleon.

2.2. Shell Model with Configuration Mixing:

In the realistic shell model, we have to take into account $\hat{H}^{(1)}$, this part of the nuclear hamiltonian that was omitted in the mean-field description. Nucleon configurations are mixed by this residual interaction. Interactions between nucleons make them jump from one orbital to another with conserve $\Gamma = (J^{\pi}, T)$, so that the wave function contains several configurations. So, we should solve the eigenvalues problem

$$\hat{H} \left| \Psi_{\Gamma} \right\rangle = E \left| \Psi_{\Gamma} \right\rangle \qquad (7)$$

Configuration mixing leads to the wave functions to consist of more than just one Slater determinant. So, we are looking for the wave function of the system in the form

$$\left| \Psi_{\Gamma} \right\rangle = \sum_{k=1}^{g} a_{k\Gamma} \left| \Phi_{k} \right\rangle \qquad (8)$$

where g the number of pure configurations considered and it is related to the valance space used, and $a_{k\Gamma}$ is amplitude(weight) the wave function $\left| \Phi_{k} \right\rangle$. Usually, the valance space incorporates all possible configurations of valence protons and valence neutrons in the partially filled orbitals, while the rest is considered as an inert core(usually, we take a double magic numbers). So we treat only the valance nucleons. This theory efficient for few numbers of valance nucleons(smaller than five valance nucleons)[8]. It is clear that the valance space becomes quickly huge for numerical treatment as the number of valance nucleons increases.

3. BCS THEORY

The BCS theory was first introduced by Bardeen, Cooper and Schrieffer in 1957 for microscopic description of the superconductivity of metals. There is experimental evidence for the presence of a similar collective condensate in atomic nuclei[6]. The valence nucleons of a nucleus feel a strong attractive force which stems from the short-range component of the nucleon–nucleon interaction. This short-range attraction was mimicked by the pure pairing force which given as[9]

$$\langle j\, j;0 | V | j\, j;0 \rangle = -\frac{1}{2} \theta^{(\ell\ell')}\, \hat{j}\, \hat{j}' G \qquad (9)$$

G is the strength of interaction, $\theta^{(\ell\ell')}$ The phase factor is chosen according to $\theta^{(\ell\ell')} = (-1)^{\ell+\ell'}$ Condon–Shortley phase convention or $\theta^{(\ell\ell')} = 1$ Biedenharn – Rose phase convention.

Having recognized the pairing phenomenon in nuclei, Bohr, Mottelson, Pines and Belyaev in 1958–59 proposed to apply BCS theory to nuclei[9]. The theory has become a standard part of the description of nuclear structure. The excited states of an even–even nucleus are created by breaking one or more pairs in the superfluid ground state $\left| \mathrm{BCS} \right\rangle$. The broken pair is interpreted as two quasiparticles, referred to as a two-quasiparticle configuration or a two-quasiparticle excitation. The energy is needed to break just one pair is called quasiparticle energy, then an extra energy equal to the binding energy of the pair has to be supplied from the outside. Thus we write BCS ground state as the vacuum for BCS quasiparticles as[9]

$$\left| \mathrm{BCS} \right\rangle = \prod_{\beta>0} u_b \sum_{N=even} \frac{1}{(N/2)!} \left| N \right\rangle \qquad (10)$$

$$\left| N \right\rangle \equiv (-\sum_{\alpha>0} \frac{v_a}{u_a} A_{\alpha}^{+})^{N/2} \left| CORE \right\rangle$$

$$\alpha = (a,\, m_a)\ ,a = (n_a,\, \ell_a,\, j_a), \beta = (b,\, m_b)\ ,b = (n_b,\, \ell_b,\, j_b)$$

$|CORE\rangle$ is the inert core, $A_\alpha^+ = c_\alpha^+ \bar{c}_\alpha^+$ the operator creates a pair of like nucleons.

c_α^+ and $\bar{c}_\alpha^+$ particle operators. The quasiparticle operators are linear combinations of particle operators via the Bogoliubov- Valatin transformation which is

$$\left.\begin{aligned} a_\alpha^+ &= u_a\, c_\alpha^+ + v_a\, \bar{c}_\alpha \\ \bar{a}_\alpha &= u_a\, \bar{c}_\alpha - v_a\, c_\alpha^+ \end{aligned}\right\} (11)$$

a_α^+ is the operator that creates a quasiparticle in orbital α, and the corresponding annihilation operator is $\bar{a}_\alpha$. In the normal BCS case each operator a_α^+ creates a quasiparticle that is a particle with probability amplitude u_a and a hole with probability amplitude v_a. This is understood so that the single-particle orbital α is empty with a probability u_a^2 and occupied with a probability v_a^2. Therefore v_a is called the occupation amplitude and u_a the un occupation amplitude of the orbital α, generically both are called occupation amplitudes. The amplitudes u_a and v_a are chosen to be real, so the normalization condition is

$$u_a^2 + v_a^2 = 1 \qquad (12)$$

The equations gives the parameters of BCS vacuum are[9]

$$\left.\begin{aligned} u_a &= \theta^{(\ell_a)} \frac{1}{\sqrt{2}} \sqrt{1+\frac{\eta_a}{E_a}}\ ,\ v_a = \frac{1}{\sqrt{2}} \sqrt{1-\frac{\eta_a}{E_a}} \\ E_a &= \sqrt{\eta_a^2 + \Delta_a^2}\ ,\ 2\,\hat{j}_a\,\Delta_a = -\sum_b \frac{\hat{j}_b}{\sqrt{\eta_a^2+\Delta_a^2}} \langle a\,a;0|V|\,b\,b;0\rangle \\ n &= \sum_a \hat{j}_a^2 v_a^2 = \frac{1}{2}\sum_a \hat{j}_a^2 \left(1-\frac{\eta_a}{E_a}\right) \end{aligned}\right\} (13)$$

The phase factor $\theta^{(\ell_a)}$ is chosen according to $\theta^{(\ell_a)} = (-1)^{\ell_a}$ Condon–Shortley phase convention or $\theta^{(\ell_a)} = 1$ Biedenharn – Rose phase convention. E_a is a quasiparticle energy, which is the least energy we needed to break a pair of nucleons, Δ_a is pairing gap, while the quantity is $E_a \geq \Delta_a$ and η_a is constant can by written in the form

$$\eta_a = \varepsilon_a - \lambda - \mu \qquad (14)$$

The quantity λ is called the chemical potential, it tells how much the energy of the BCS ground state grows when one particle is added to it. The quantity μ_a is called the self-energy, It describes the fact that the energy of a nucleon in orbital a gets additional contributions from its interactions with the other nucleons, may be written in the form

$$\mu_a = -\hat{j}_a^{-2} \sum_{bJ} v_b^2 \hat{J}^2 [\mathrm{N}_{ab}(J)]^{-2} \langle a\,b;J|V|\,a\,b;J\rangle \quad (15)$$

Where $\mathrm{N}_{ab}(J)$ is normalization factor. These equations crystallize the information contained in the BCS framework. They must be solved numerically, which requires iterative methods.

4. MIXING OF TWO-QUASIPARTICLE CONFIGURATIONS

When we take into account $\hat{H}^{(1)}$ between two quasiparticle, it make them jump from one orbital to another without change Γ, just like nucleons. So that, the total wave function contains several configurations from wave functions $|\mathrm{BCS}\rangle_1, |\mathrm{BCS}\rangle_2, |\mathrm{BCS}\rangle_3 \ldots$

The total wave function for quasiparticle is thus given as

$$\left.\begin{aligned}|\omega\rangle = Q_\omega^+|\mathrm{BCS}\rangle = \sum_{a\le b} \mathrm{X}_{ab}^\omega A_{ab}^+(JM)|\mathrm{BCS}\rangle \\ |\omega\rangle = \sum_{ab} \mathrm{X}_{ab}^\omega |a\ b, J\rangle\end{aligned}\right\} (16)$$

where $\omega = n J^\pi M$ is the full set of quantum numbers(principles quantum number and total angular momentum and its projection) and X_{ab}^ω is amplitude for the wave function $|ab, J\rangle$. This is excitation is often called a phonon, the term refers to the collective properties. First we write the projection of the eigenvalue equation(7) onto the basis state $|\mathrm{BCS}\rangle$ and then expand it by using(16) as

$$(E_a + E_b)\mathrm{X}_{ab}^\omega + \sum_{c\le d} \mathrm{X}_{cd}^\omega \langle \mathrm{BCS}|[A_{ab}(J\,M), V, A_{cd}^+]|\mathrm{BCS}\rangle = E_\omega \mathrm{X}_{ab}^\omega \quad (17)$$

Which we can write

$$\sum_{c\le d} A_{ab,cd}\mathrm{X}_{cd}^\omega = E_\omega \mathrm{X}_{ab}^\omega \quad (18)$$

This equation written as a matrix equation

$$\mathrm{A}\,\mathrm{X}^\omega = E_\omega \mathrm{X}^\omega \quad (19)$$

The quantities $A_{ab,cd}$ are the elements of the matrix A is given as[9]

$$\begin{aligned}A_{ab,cd} &= (E_a + E_b)\delta_{ac}\delta_{bd} + (u_a u_b u_c u_d + v_a v_b v_c v_d)\langle ab; J|V|cd; J\rangle \\ &+ N_{ab}(J)N_{cd}(J)[(u_a v_b u_c v_d + v_a u_b v_c u_d)\langle ab^{-1}; J|V_{res}|cd^{-1}; J\rangle \\ &-(-1)^{j_c+j_d+J}(u_a v_b v_c u_d + v_a u_b u_c v_d)\langle ab^{-1}; J|V_{res}|dc^{-1}; J\rangle] \quad (20)\end{aligned}$$

where $\langle ab^{-1}; J|V_{res}|dc^{-1}; J\rangle$ is a particle–hole matrix elements, which it follows from the Pandya transformation[9]

$$\langle ab^{-1}; J|V_{res}|cd^{-1}; J\rangle = -\sum_{J'} \hat{J}'^2 \begin{Bmatrix} j_a & j_b & J \\ j_c & j_d & J'\end{Bmatrix}\langle ad; J'|V|cb; J'\rangle \quad (21)$$

For pure configurations, the equation(20) became as

$$\begin{aligned}A_{ab,ab} &= (E_a + E_b) + (u_a^2 u_b^2 + v_b^2 v_a^2)\langle ab; J|V|cd; J\rangle \\ &+ N_{ab}(J)N_{cd}(J)[(u_a^2 v_b^2 + v_a^2 u_b^2)\langle ab^{-1}; J|V_{res}|cd^{-1}; J\rangle \\ &-(-1)^{j_c+j_d+J}(u_a v_b v_c u_d + v_a u_b u_c v_d)\langle ab^{-1}; J|V_{res}|dc^{-1}; J\rangle] \quad (22)\end{aligned}$$

Since the quasiparticle indices carry also the nucleon kind, the formalism for proton–neutron excitations can be obtained as a modification of the above, equation(20) is modified to reads as

$$\begin{aligned}A_{pn,p'n'} &= (E_p + E_n)\delta_{pp'}\delta_{nn'} + (u_p u_n u_{p'} u_{n'} + v_p v_n v_{p'} v_{n'})\langle pn; J|V|p'n'; J\rangle \\ &+ (u_p v_n u_{p'} v_{n'} + v_p u_n v_{p'} u_{n'})\langle pn^{-1}; J|V_{res}|p'n'^{-1}; J\rangle \quad (23)\end{aligned}$$

The Pandya transformation (21) is

now $$\langle pn^{-1}; J|V_{res}|p'n'^{-1}; J\rangle = -\sum_{J'} \hat{J}'^2 \begin{Bmatrix} j_p & j_n & J \\ j_{p'} & j_{n'} & J'\end{Bmatrix}\langle pn; J'|V|p'n'; J'\rangle \quad (24)$$

For the pure configurations in proton- neutron formalism. Hence, the equation(23) became as

$$\begin{aligned}A_{pn,pn} &= (E_p + E_n) + (u_p^2 u_n^2 + v_p^2 v_n^2)\langle pn; J|V|pn; J\rangle \\ &+ (u_p^2 v_n^2 + u_n^2 v_p^2)\langle pn^{-1}; J|V_{res}|p\ n^{-1}; J\rangle \quad (25)\end{aligned}$$

So, we have converted the Schrodinger equation for quasiparticle into an eigenvalues problem of the hamiltonian matrix. The eigenvalues and eigen states of a general hamiltonian are obtained by diagonalizing this hamiltonian matrix.

5. THE MODIFIED SURFACE DELTA INTERACTION (MSDI)

Residual nucleon-nucleon interaction is the part of the interaction which is not include in the central average potential. In consequence of the Pauli principle prohibit most inelastic collisions in the nuclear matter, Therefore nucleons move freely in the nucleus and the effective interactions between the nucleons occurs mainly in the nuclear surface. The simplest form of residual interaction which agree with this description is the Surface Delta Interactions(SDI)[2]. Which considered separable interaction, this property greatly simplifies the calculation of two-body matrix element. This form was proposed in 1966 by Moszkowski and co-workers[8], and developed by Glaudemans who added the isospin dependence in the matrix element of the two body interaction, and it called the MSDI, the interaction may be written in the form[10]

$$V_{ab}[\text{MSDI}] = -4\pi A_T \,\delta(\Omega_{ab})\delta(\hat{r}_a - \hat{R})\delta(\hat{r}_b - \hat{R}) + B_T \hat{t}_a \hat{t}_b \quad (26)$$

Where A_T and B_T are the strength of interaction, the values of this terms determine by means of "Least-Square Fitting", Ω_{ab} is the angular distance between the interacting particles, $\hat{r}_a$, $\hat{r}_b$ are the position vectors of interacting particles, $\hat{R}$ is the nuclear radius and $\hat{t}$ is isospin operator. The two-body matrix elements of the MSDI are given as[10]

$$\left\langle j_a j_b, JT \middle| V_{MSDI} \middle| j_c j_d, JT \right\rangle_{JT} = \frac{A_T}{2(2J+1)}\left[\frac{(2j_a+1)(2j_b+1)(2j_c+1)(2j_d+1)}{(1+\delta_{ab})(1+\delta_{cd})}\right]^{\frac{1}{2}} \{(-1)^{\ell_a+\ell_c+j_b+j_d}\left\langle j_b -\frac{1}{2}\, j_a \,\frac{1}{2}\middle| J0\right\rangle$$ Correspondingly, J and T are

$$\left\langle j_d -\frac{1}{2}\, j_c \,\frac{1}{2}\middle| J0\right\rangle[1-(-1)^{\ell_c+\ell_d+J+T}] - \left\langle j_b \,\frac{1}{2}\, j_a \,\frac{1}{2}\middle| J1\right\rangle\left\langle j_d \,\frac{1}{2}\, j \,\frac{1}{2}\middle| J1\right\rangle[1+(-1)^T]\} + B_T[2T(T+1)-3]\delta_{ac}\delta_{bd} \quad (27)$$

the spin and isospin of the two-particle system. In the case of pure configurations, the last expression became as

$$\left\langle j_a\, j_b, JT \middle| V_{MSDI} \middle| j_a\, j_b, JT \right\rangle_{JT} = -A_T \frac{(2j_a+1)(2j_b+1)}{2(2J+1)(1+\delta_{ab})}\{\left\langle j_a \,\frac{1}{2}\, j_b -\frac{1}{2}\middle| J0\right\rangle^2[1-(-1)^{\ell_a+\ell_b+J+T}] +$$

$$\left\langle j_a \,\frac{1}{2}\, j_b \,\frac{1}{2}\middle| J1\right\rangle^2[1+(-1)^T]\} + B_T[2T(T+1)-3] \quad (28)$$

6. ELECTROMAGNETIC TRANSITIONS

The transition probability represent a sensitive test for the most modern effective interactions that have been developed to describe $1p\text{-}0f\text{-}0g_{9/2}$ shell nuclei.

Consider the transition probability per unit time, usually called just transition probability of gamma decay from an initial nuclear state i to a final nuclear state f denoted T_{fi} and its half-life is

$$t_{1/2} = \frac{\ln 2}{T_{fi}} \quad (29)$$

The transition probability calculated by the 'golden rule' of time-dependent perturbation theory, as[11]

$$T_{fi}^{(\sigma\lambda\mu)} = \frac{2}{\varepsilon_0 \hbar}\frac{\lambda+1}{\lambda[(2\lambda+1)!!]^2}\left(\frac{E_\gamma}{\hbar c}\right)^{2\lambda+1}\left|\left\langle \varsigma_f J_f m_f \,|\, \hat{M}_{\sigma\lambda\mu} \,|\, \varsigma_i J_i m_i \right\rangle\right|^2 \quad (30)$$

Where $\hbar = \frac{h}{2\pi}$ and h is blank constant, c is speed of light of vacuum and ε_0 is the electric capacity of vacuum. E_γ is the energy of the transition, $\hat{M}_{\sigma\lambda\mu}$ is the nuclear operator associated with the multiple radiation field λ, ς is the full set of quantum numbers which we need to define the states, The sources of the field are either of electric or magnetic type, designated by an index σ such that $\sigma = E$ or $\sigma = \mathrm{M}$ so $\hat{M}_{E\lambda} = \hat{Q}_\lambda$ or $\hat{M}_{\mathrm{M}\lambda} = \hat{\mathrm{M}}_\lambda$. Where we have the reduced transition probability is[11]

$$B(\sigma\lambda:\varsigma_i J_i \to \varsigma_f J_f) = \frac{1}{2J_i+1}\left|(\varsigma_f J_f \| \hat{M}_{\sigma\lambda} \| \varsigma_i J_i)\right|^2 \quad (31)$$

$(\varsigma_f J_f \| \hat{M}_{\sigma\lambda} \| \varsigma_i J_i)$ is a reduced matrix elements. We can write the equation(30) for electric and magnetic transition as

$$\left.\begin{aligned} T_{fi}^{E\lambda} &= \alpha\hbar c \frac{8\pi(\lambda+1)}{\lambda[(2\lambda+1)!!]^2}\frac{1}{\hbar}\left(\frac{1}{\hbar c}\right)^{2\lambda+1} E_\gamma^{2\lambda+1} B(E\lambda) \quad [e^2\ fm^{2\lambda}]1/\sec \\ T_{fi}^{M\lambda} &= \alpha\hbar c\left(\frac{\hbar c}{2m_p c^2}\right)^2 \frac{8\pi(\lambda+1)}{\lambda[(2\lambda+1)!!]^2}\frac{1}{\hbar}\left(\frac{1}{\hbar c}\right)^{2\lambda+1} E_\gamma^{2\lambda+1} B(M\lambda) \quad [(\mu_N/c)^2\ fm^{2\lambda-2}]1/\sec \end{aligned}\right\} (32)$$

α here is the hyper fine constant and μ_N is the nuclear magneton.

We can write the reduced single-particle matrix element for electric transitions as[9]

$$(a\|\hat{Q}_\lambda\|b) = \theta_{ab}^{(E\lambda)}\frac{e}{\sqrt{4\pi}}(-1)^{j_a+\lambda-\frac{1}{2}}\frac{1+(-1)^{\ell_a+\ell_b+\lambda}}{2}\hat{\lambda}\,\hat{j}_a\hat{j}_b\begin{pmatrix} j_a & j_b & \lambda \\ \frac{1}{2} & -\frac{1}{2} & 0\end{pmatrix}\Re_{ab}^{(\lambda)} \quad (33)$$

e is the nucleon charge, the phase factor $\theta_{ab}^{(E\lambda)}$ is chosen according to $\theta_{ab}^{(E\lambda)}=1$ Condon–Shortley phase convention or $\theta_{ab}^{(E\lambda)}=(-1)^{0.5(\ell_b-\ell_a+\lambda)}$ Biedenharn –Rose phase convention, $\Re_{ab}^{(\lambda)}$ is define radial integrals for harmonic oscillator[9]. The single-particle matrix element for magnetic transitions is[9]

$$\left.\begin{aligned} (a\|\hat{M}_\lambda\|b) &= \theta_{ab}^{(M\lambda)}\frac{\mu_N/c}{\sqrt{4\pi}}(-1)^{j_b+\lambda-\frac{1}{2}}\frac{1-(-1)^{\ell_a+\ell_b+\lambda}}{2}\hat{\lambda}\,\hat{j}_a\hat{j}_b\begin{pmatrix} j_a & j_b & \lambda \\ \frac{1}{2} & -\frac{1}{2} & 0\end{pmatrix}\times(\lambda-k)[g_\ell(1+\frac{k}{\lambda+1})-\frac{1}{2}g_s]\Re_{ab}^{(\lambda-1)} \\ k &\equiv (-1)^{\ell_a+j_a+\frac{1}{2}}(j_a+\frac{1}{2})+(-1)^{\ell_b+j_b+\frac{1}{2}}(j_b+\frac{1}{2}) \end{aligned}\right\} (34)$$

The phase factor $\theta_{ab}^{(M\lambda)}$ is chosen according to $\theta_{ab}^{(M\lambda)}=1$ Condon–Shortley phase convention or $\theta_{ab}^{(M\lambda)}=(-1)^{0.5(\ell_b-\ell_a+\lambda+1)}$ Biedenharn – Rose phase convention, and g is the gyromagnetic ratios.

First we begin with the electromagnetic transition of excited states for an even–even nucleus to its BCS ground state. These excited states are proton - proton and neutron–neutron two-quasiparticle states, which given by using the amplitude[11]

$$\left(\mathrm{BCS}\|M_{\sigma\lambda}\|J^\pi\right) = \delta_{\lambda J}\sum_{a\le b}\mathrm{X}_{ab}^{J^\pi}N_{ab}(J)\,\theta^{l_b}(v_a|u_b|\pm v_b|u_a|)\left(a\|M_{\sigma\lambda}\|b\right) \quad (35)$$

Where (+) for $\sigma=E$, (-) for $\sigma=\mathrm{M}$. The phase factor θ^{l_b} is defined in(13).

For an odd–odd nucleus the initial and final states are written as proton-neutron two-quasiparticle states. The transition amplitude for these states can be written in the final form as[9]

$$(p_f n_f; J_f\|\hat{M}_{\sigma\lambda}\|p_i n_i; J_i) = \hat{J}_i\hat{J}_f[\delta_{n_i n_f}(-1)^{j_{p_f}+j_{n_f}+J_i+\lambda}\begin{Bmatrix} J_f & J_i & \lambda \\ j_{p_i} & j_{p_f} & j_{n_f}\end{Bmatrix}D_{p_i p_f}^{(\lambda)}(p_f\|\hat{M}_{\sigma\lambda}\|p_i)$$

$$+\delta_{p_i p_f}(-1)^{j_{p_i}+j_{n_i}+J_f+\lambda}\begin{Bmatrix} J_f & J_i & \lambda \\ j_{n_i} & j_{n_f} & j_{p_f}\end{Bmatrix}D_{n_i n_f}^{(\lambda)}(n_f\|\hat{M}_{\sigma\lambda}\|n_i)] \quad (36)$$

7. CONFIGURATION MIXING FOR ELECTROMAGNETIC TRANSITIONS

The wave function of BCS excited state is a linear combination of two quasiparticle components which given by equation (16). So, the initial and final BCS wave functions is

$$\left.\begin{aligned} |\omega_i\rangle &= Q_{\omega_i}^+|\mathrm{BCS}\rangle = \sum_{a_i\le b_i}\mathrm{X}_{a_i b_i}^{\omega_i}A_{a_i b_i}^+(J_i M_i)|\mathrm{BCS}\rangle \\ |\omega_f\rangle &= Q_{\omega_f}^+|\mathrm{BCS}\rangle = \sum_{a_f\le b_f}\mathrm{X}_{a_f b_f}^{\omega_f}A_{a_f b_f}^+(J_f M_f)|\mathrm{BCS}\rangle \end{aligned}\right\} (37)$$

The decay amplitude between these states is

$$\left\langle \omega_i \| \hat{M}_{\sigma\lambda} \| \omega_f \right\rangle_{\text{BCS}} = \sum_{\substack{a_i \le b_i \\ a_f \le b_f}} \mathrm{X}^{\omega_f}_{a_f b_f} \mathrm{X}^{\omega_i}_{a_i b_i} (J_i M_i)(a_f b_f ; J_f \| \hat{M}_{\sigma\lambda} \| a_i b_i ; J_i) \quad (38)$$

The two quasiparticle matrix element- which appear on the right hand side- it takes the matrix element for electromagnetic transitions.

8. CALCULATIONS AND DISCUSSIONS

In this work, the single-particle states are chosen for both protons and neutrons in MeV as[9]

$\varepsilon_{0f_{7/2}} = 0,\ \varepsilon_{1p_{3/2}} = 4.80,\ \varepsilon_{0f_{5/2}} = 8.40,\ \varepsilon_{1p_{1/2}} = 6.82,\ \varepsilon_{0g_{9/2}} = 8.80$

We take $^{40}_{20}Ca$ as an inert core for three nuclei which are $^{78}_{34}Se, ^{78}_{35}Br$ and $^{78}_{36}Kr$, and we restrict ourselves in the full space $1p\text{-}0f\text{-}0g_{9/2}$ including five single-particle orbits which are $0f_{7/2}, 1p_{3/2}, 0f_{5/2}, 1p_{1/2}$ and $0g_{9/2}$ to compute first excited states, and to compute $B(M\lambda)$ values and half-lives for electromagnetic transitions. We use the shell model code AUSM[7]to calculate all these values.

8.1. Excited states:

For $^{78}_{34}Se$ nucleus, the best value for the MSDI strengths given as

$A_T = 0.53(\mathrm{MeV}), B_T = 0.5(\mathrm{MeV})$ [12] which can be determined by a least-squares fitting procedure. For $^{78}_{35}Br$ the best values for the MSDI strengths given as $A_T = 0.34(\mathrm{MeV}), B_T = -0.95(\mathrm{MeV})$ and For $^{78}_{36}Kr$ the MSDI strengths given as $A_T = 0.5(\mathrm{MeV}), B_T = 0.42(\mathrm{MeV})$ [12]. Table 1, 2 and 3 represents the comparison between our theoretical calculations for the first excited states in MeV for pure configuration and for configuration mixing using the residual effective interactions MSDI with the experimental data taken from Ref.[13,14] and the total wave functions for $^{78}_{34}Se, ^{78}_{35}Br$ and $^{78}_{36}Kr$ nuclei, respectively.

Table 1. calculated values for first excited states in the pure configuration and in the configuration mixing compared with the experimental values for $^{78}_{34}Se$

J^π	E^{PC}_{th}	E^{CM}_{th}	$E_{exp.}$	$\lvert\omega_i\rangle = \sum_{a_i b_i} \mathrm{X}^{\omega_i}_{a_i b_i} \lvert a_i\, b_i, J_i\rangle$
0^+	0	0	0	+ 0.5947 $\lvert\pi\, 0f_{7/2}\pi\, 0f_{7/2}; 0^+\rangle$ + 0.1939 $\lvert\pi 1p_{3/2}\pi 1p_{3/2}; 0^+\rangle$ + 0.1319 $\lvert\pi\, 0f_{5/2}\pi\, 0f_{5/2}; 0^+\rangle$ + 0.0660 $\lvert\pi 1p_{1/2}\pi 1p_{1/2}; 0^+\rangle$ - 0.2948 $\lvert\pi 0g_{9/2}\pi 0g_{9/2}; 0^+\rangle$ + 0.5947 $\lvert\nu 0f_{7/2}\nu 0f_{7/2}; 0^+\rangle$ + 0.1939 $\lvert\nu 1p_{3/2}\nu 1p_{3/2}; 0^+\rangle$ + 0.1319 $\lvert\nu 0f_{5/2}\nu 0f_{5/2}; 0^+\rangle$ + 0.0660 $\lvert\nu 1p_{1/2}\nu 1p_{1/2}; 0^+\rangle$ - 0.2948 $\lvert\nu 0g_{9/2}\nu 0g_{9/2}; 0^+\rangle$
2^+	2.3410	1.3205	0.6137	+ 0.0147 $\lvert\pi\, 0f_{7/2}\pi\, 0f_{7/2}; 2^+\rangle$ - 0.0338 $\lvert\pi\, 0f_{7/2}\pi\, 1p_{3/2}; 2^+\rangle$ + 0.0005 $\lvert\pi\, 0f_{7/2}\pi\, 0f_{5/2}; 2^+\rangle$ + 0.0230 $\lvert\pi\, 1p_{3/2}\pi\, 1p_{3/2}; 2^+\rangle$ - 0.1576 $\lvert\pi\, 1p_{3/2}\pi\, 1p_{1/2}; 2^+\rangle$ - 0.0092 $\lvert\pi\, 0f_{5/2}\pi\, 1p_{3/2}; 2^+\rangle$ + 0.1894 $\lvert\pi\, 0f_{5/2}\pi\, 0f_{5/2}; 2^+\rangle$ + 0.6609 $\lvert\pi\, 0f_{5/2}\pi\, 1p_{1/2}; 2^+\rangle$ - 0.0246 $\lvert\pi\, 0g_{9/2}\pi\, 0g_{9/2}; 2^+\rangle$ + 0.0147 $\lvert\nu\, 0f_{7/2}\nu\, 0f_{7/2}; 2^+\rangle$ - 0.0338 $\lvert\nu\, 0f_{7/2}\nu\, 1p_{3/2}; 2^+\rangle$ + 0.0005 $\lvert\nu\, 0f_{7/2}\nu\, 0f_{5/2}; 2^+\rangle$ + 0.0230 $\lvert\nu\, 1p_{3/2}\nu\, 1p_{3/2}; 2^+\rangle$ - 0.1576 $\lvert\nu\, 1p_{3/2}\nu\, 1p_{1/2}; 2^+\rangle$ - 0.0092 $\lvert\nu\, 0f_{5/2}\nu\, 1p_{3/2}; 2^+\rangle$ + 0.1894 $\lvert\nu\, 0f_{5/2}\nu\, 0f_{5/2}; 2^+\rangle$ + 0.6609 $\lvert\nu\, 0f_{5/2}\nu\, 1p_{1/2}; 2^+\rangle$ - 0.0246 $\lvert\nu\, 0g_{9/2}\nu\, 0g_{9/2}; 2^+\rangle$
4^+	2.6256	1.8023	1.5028	- 0.6676 $\lvert\pi\, 0f_{7/2}\pi\, 0f_{7/2}; 4^+\rangle$ + 0.1839 $\lvert\pi\, 0f_{7/2}\pi\, 1p_{3/2}; 4^+\rangle$ + 0.0678 $\lvert\pi\, 0f_{7/2}\pi\, 0f_{5/2}; 4^+\rangle$ - 0.0784 $\lvert\pi\, 0f_{7/2}\pi\, 1p_{1/2}; 4^+\rangle$ + 0.0924 $\lvert\pi\, 0f_{5/2}\pi\, 1p_{3/2}; 4^+\rangle$ - 0.0261 $\lvert\pi\, 0f_{5/2}\pi\, 0f_{5/2}; 4^+\rangle$ + 0.0245 $\lvert\pi\, 0g_{9/2}\pi\, 0g_{9/2}; 4^+\rangle$ - 0.6676 $\lvert\nu\, 0f_{7/2}\nu\, 0f_{7/2}; 4^+\rangle$ + 0.1839 $\lvert\nu\, 0f_{7/2}\nu\, 1p_{3/2}; 4^+\rangle$ + 0.0678 $\lvert\nu\, 0f_{7/2}\nu\, 0f_{5/2}; 4^+\rangle$ - 0.0784 $\lvert\nu\, 0f_{7/2}\nu\, 1p_{1/2}; 4^+\rangle$ + 0.0924 $\lvert\nu\, 0f_{5/2}\nu\, 1p_{3/2}; 4^+\rangle$ - 0.0261 $\lvert\nu\, 0f_{5/2}\nu\, 0f_{5/2}; 4^+\rangle$ + 0.0245 $\lvert\nu\, 0g_{9/2}\nu\, 0g_{9/2}; 4^+\rangle$
6^+	2.7185	2.0179	2.5465	- 0.7043 $\lvert\pi\, 0f_{7/2}\pi\, 0f_{7/2}; 6^+\rangle$ + 0.0623 $\lvert\pi\, 0f_{7/2}\pi\, 0f_{5/2}; 6^+\rangle$ + 0.0107 $\lvert\pi\, 0g_{9/2}\pi\, 0g_{9/2}; 6^+\rangle$ - 0.7043 $\lvert\nu\, 0f_{7/2}\pi\, 0f_{7/2}; 6^+\rangle$ + 0.0623 $\lvert\nu\, 0f_{7/2}\nu\, 0f_{5/2}; 6^+\rangle$ + 0.0107 $\lvert\nu\, 0g_{9/2}\nu\, 0g_{9/2}; 6^+\rangle$
8^+	2.7836	4.6537	3.5850	- 0.7071 $\lvert\pi\, 0g_{9/2}\pi\, 0g_{9/2}; 8^+\rangle$ - 0.7071 $\lvert\nu\, 0g_{9/2}\nu\, 0g_{9/2}; 8^+\rangle$

From table 1, we found the average absolute deviation between theoretical calculations and experimental data in the pure configuration is $\Delta\overline{E}^{PC}=\dfrac{\sum_{i=1}^{N}\left|E_{\exp}(i)-E_{cal}(i)\right|}{N}=0.7647(\text{MeV})$ (here N is the number of excited states) for $^{78}_{34}Se$, this average became in the states of configuration mixing for all allowed states on the full space $1p\text{-}0f\text{-}0g_{9/2}$ is $\Delta\overline{E}^{CM}=0.5207$ (MeV) for this nucleus.

Table 2. calculated values for first excited states in the pure configuration and in the configuration mixing compared with the experimental values for $^{78}_{35}Br$

J^{π}	E_{th}^{PC}	E_{th}^{CM}	$E_{\exp.}$	$\omega_i\rangle=\sum_{p_i n_i}\mathrm{X}^{\omega_i}_{p_i n_i}\left\|p_i\ n_i, J_i\right\rangle$
1^+	0	0	0	$+0.3622\ \left\|\pi 0f_{7/2}\nu 0f_{7/2};1^-\right\rangle-0.5005\ \left\|\pi 0f_{7/2}\nu 0f_{5/2};1^-\right\rangle$ $+0.3109\ \left\|\pi 1p_{3/2}\nu 1p_{3/2};1^-\right\rangle-0.2966\ \left\|\pi 1p_{3/2}\nu 1p_{1/2};1^-\right\rangle$ $+0.1574\ \left\|\pi 0f_{5/2}\nu 1p_{3/2};1^-\right\rangle-0.4255\ \left\|\pi 0f_{5/2}\nu 0f_{5/2};1^-\right\rangle$ $-0.2987\ \left\|\pi 1p_{1/2}\nu 1p_{1/2};1^-\right\rangle-0.3723\ \left\|\pi 0g_{9/2}\nu 0g_{9/2};1^-\right\rangle$
2^+	0.0887	0.2658	0.0197	$-0.3111\left\|\pi 0f_{7/2}\nu 0f_{7/2};2^-\right\rangle+0.4583\ \left\|\pi 0f_{7/2}\nu 1p_{3/2};2^-\right\rangle$ $+0.1366\left\|\pi 0f_{7/2}\nu 0f_{5/2};2^-\right\rangle-0.2747\ \left\|\pi 1p_{3/2}\nu 1p_{3/2};2^-\right\rangle$ $+0.3616\left\|\pi 1p_{3/2}\nu 1p_{1/2};2^-\right\rangle+0.2757\ \left\|\pi 0f_{5/2}\nu p_{3/2};2^-\right\rangle$ $-0.3117\left\|\pi 0f_{5/2}\nu 0f_{5/2};2^-\right\rangle+0.4105\ \left\|\pi 0f_{5/2}\nu 1p_{3/2};2^-\right\rangle$ $+0.3559\left\|\pi 0g_{9/2}0g_{9/2};2^-\right\rangle$
4^+	0.2185	0.3386	0.1809	$-0.4829\left\|\pi 0f_{7/2}\nu 0f_{7/2};4^-\right\rangle+0.4603\left\|\pi 0f_{7/2}\nu 1p_{3/2};4^-\right\rangle$ $+0.3251\left\|\pi 0f_{7/2}\nu 0f_{5/2};4^-\right\rangle-0.3178\left\|\pi 0f_{7/2}\nu 1p_{1/2};4^-\right\rangle$ $+0.5173\left\|\pi 0f_{5/2}\nu p_{3/2};4^-\right\rangle-0.2766\left\|\pi 0f_{5/2}\nu 0f_{5/2};4^-\right\rangle$ $+0.0649\left\|\pi 0g_{9/2}0g_{9/2};4^-\right\rangle$
3^+	0.4341	1.0175	1.0260	$0.4234\left\|\pi 0f_{7/2}\nu 0f_{7/2};3^-\right\rangle-0.3880\left\|\pi 0f_{7/2}\nu 1p_{3/2};3^-\right\rangle$ $0.0617\left\|\pi 0f_{7/2}\nu 0f_{5/2};3^-\right\rangle+0.2178\left\|\pi 0f_{7/2}\nu p_{1/2};3^-\right\rangle$ $0.4140\left\|\pi 1p_{3/2}\nu 1p_{3/2};3^-\right\rangle-0.0913\left\|\pi 0f_{5/2}\nu p_{1/2};3^-\right\rangle$ $0.3906\left\|\pi 0f_{5/2}\nu 0f_{5/2};3^-\right\rangle+0.5270\left\|\pi 0f_{5/2}\nu 1p_{3/2};3^-\right\rangle$ $0.0047\left\|\pi 0g_{9/2}0g_{9/2};3^-\right\rangle$

And we found from the table 2 the average absolute deviation between theoretical calculations and experimental data in the pure configuration for $^{78}_{35}Br$ is $\Delta\overline{E}^{PC}=0.1746(\text{MeV})$, this average became in the states of configuration mixing is $\Delta\overline{E}^{CM}=0.1031$ (MeV).

Table 3. calculated values for first excited states in the pure configuration and in the configuration mixing compared with the experimental values for $^{78}_{36}Kr$

J^{π}	E_{th}^{PC}	E_{th}^{CM}	$E_{\exp.}$	$\left\|\omega_i\right\rangle=\sum_{a_i b_i}\mathrm{X}^{\omega_i}_{a_i b_i}\left\|a_i\ b_i, J_i\right\rangle$
0^+	0	0	0	$+0.5929\ \left\|\pi 0f_{7/2}\pi 0f_{7/2};0^+\right\rangle+0.1950\ \left\|\pi 1p_{3/2}\pi 1p_{3/2};0^+\right\rangle$ $+0.1333\ \left\|\pi 0f_{5/2}\pi 0f_{5/2};0^+\right\rangle+0.0666\ \left\|\pi 1p_{1/2}\pi 1p_{1/2};0^+\right\rangle$ $-0.2970\ \left\|\pi 0g_{9/2}\pi 0g_{9/2};0^+\right\rangle+0.5929\ \left\|\nu 0f_{7/2}\nu 0f_{7/2};0^+\right\rangle$ $+0.1950\ \left\|\nu 1p_{3/2}\nu 1p_{3/2};0^+\right\rangle+0.1333\ \left\|\nu 0f_{5/2}\nu 0f_{5/2};0^+\right\rangle$ $+0.0666\ \left\|\nu 1p_{1/2}\nu 1p_{1/2};0^+\right\rangle-0.2970\ \left\|\nu 0g_{9/2}\nu 0g_{9/2};0^+\right\rangle$
2^+	2.1090	1.3783	0.4550	$-0.0152\ \left\|\pi 0f_{7/2}\pi 0f_{7/2};2^+\right\rangle+0.0357\ \left\|\pi 0f_{7/2}\pi 1p_{3/2};2^+\right\rangle$ $-0.0004\ \left\|\pi 0f_{7/2}\pi 0f_{5/2};2^+\right\rangle-0.0229\ \left\|\pi 1p_{3/2}\pi 1p_{3/2};2^+\right\rangle$ $+0.1452\ \left\|\pi 1p_{3/2}\pi 1p_{1/2};2^+\right\rangle+0.0150\ \left\|\pi 0f_{5/2}\pi 1p_{3/2};2^+\right\rangle$ $-0.1832\ \left\|\pi 0f_{5/2}\pi 0f_{5/2};2^+\right\rangle-0.6652\ \left\|\pi 0f_{5/2}\pi 1p_{1/2};2^+\right\rangle$ $+0.0259\ \left\|\pi 0g_{9/2}\pi 0g_{9/2};2^+\right\rangle-0.0152\ \left\|\nu 0f_{7/2}\nu 0f_{7/2};2^+\right\rangle$ $+0.0357\ \left\|\nu 0f_{7/2}\nu 1p_{3/2};2^+\right\rangle-0.0004\ \left\|\nu 0f_{7/2}\nu 0f_{5/2};2^+\right\rangle$ $-0.0229\ \left\|\nu 1p_{3/2}\nu 1p_{3/2};2^+\right\rangle+0.1452\ \left\|\nu 1p_{3/2}\nu 1p_{1/2};2^+\right\rangle$ $+0.0150\ \left\|\nu 0f_{5/2}\nu 1p_{3/2};2^+\right\rangle-0.1832\ \left\|\nu 0f_{5/2}\nu 0f_{5/2};2^+\right\rangle$ $-0.6652\ \left\|\nu 0f_{5/2}\nu 1p_{1/2};2^+\right\rangle+0.0259\ \left\|\nu 0g_{9/2}\nu 0g_{9/2};2^+\right\rangle$
4^+	2.3045	1.8382	1.1195	$-0.0122\ \left\|\pi 0f_{7/2}\pi 0f_{7/2};4^+\right\rangle+0.0146\ \left\|\pi 0f_{7/2}\pi 1p_{3/2};4^+\right\rangle$ $+0.0173\ \left\|\pi 0f_{7/2}\pi 0f_{5/2};4^+\right\rangle-0.0212\ \left\|\pi 0f_{7/2}\pi 1p_{1/2};4^+\right\rangle$ $+0.0424\ \left\|\pi 0f_{5/2}\pi 1p_{3/2};4^+\right\rangle+0.7048\ \left\|\pi 0f_{5/2}\pi 0f_{5/2};4^+\right\rangle$ $+0.0179\ \left\|\pi 0g_{9/2}\pi 0g_{9/2};4^+\right\rangle-0.0122\ \left\|\nu 0f_{7/2}\nu 0f_{7/2};4^+\right\rangle$ $+0.0146\ \left\|\nu 0f_{7/2}\nu 1p_{3/2};4^+\right\rangle+0.0173\ \left\|\nu 0f_{7/2}\nu 0f_{5/2};4^+\right\rangle$ $-0.0212\ \left\|\nu 0f_{7/2}\nu 1p_{1/2};4^+\right\rangle+0.0424\ \left\|\nu 0f_{5/2}\nu 1p_{3/2};4^+\right\rangle$ $+0.7048\ \left\|\nu 0f_{5/2}\nu 0f_{5/2};4^+\right\rangle+0.0179\ \left\|\nu 0g_{9/2}\nu 0g_{9/2};4^+\right\rangle$
6^+	2.3696	2.0139	1.9778	$+0.7044\ \left\|\pi 0f_{7/2}\pi 0f_{7/2};6^+\right\rangle-0.0609\ \left\|\pi 0f_{7/2}\pi 0f_{5/2};6^+\right\rangle$ $-0.0119\ \left\|\pi 0g_{9/2}\pi 0g_{9/2};6^+\right\rangle+0.7044\ \left\|\nu 0f_{7/2}\pi 0f_{7/2};6^+\right\rangle$ $-0.0609\ \left\|\nu 0f_{7/2}\nu 0f_{5/2};6^+\right\rangle-0.0119\ \left\|\nu 0g_{9/2}\nu 0g_{9/2};6^+\right\rangle$
8^+	2.4168	4.6488	2.9933	$-0.7071\ \left\|\pi 0g_{9/2}\pi 0g_{9/2};8^+\right\rangle-0.7071\ \left\|\nu 0g_{9/2}\nu 0g_{9/2};8^+\right\rangle$

and we found from the table 3 the average absolute deviation between theoretical calculations and experimental data in the pure configuration is $\Delta\overline{E}^{PC}=0.7615(\text{MeV})$ for $^{78}_{36}Kr$, this average became in the states of configuration mixing is $\Delta\overline{E}^{CM}=0.6667(\text{MeV})$.

So, taking configuration mixing into consideration enhanced considerably the results for first excited states more than the case of pure configuration when comparison with the experimental data.

8.2. Electromagnetic Transitions and $B(M\lambda)$ Values:

Table 4,5 and 6 represents the comparison between our theoretical calculations for the transition amplitude, reduced transition probability, transition probability and half-lives for pure configuration and for

configuration mixing with the data taken from Ref.[13,14].

Table 4. calculated values of transition amplitude, reduced transition probability, transition probability and half-live in the pure configuration and in the configuration mixing compared with the experimental value for $^{78}_{34}Se$

CONFIGURATION	$(\text{BCS}\|\|Q_2\|\|ab;2^+)$ (efm^2)	$B(E2:2^+\rightarrow 0^+_{gs})$ (e^2fm^4)	$T(E2)$ $(1/\text{sec})$	$t_{1/2}^{th.}$ (ps)	$t_{1/2}^{\exp.}$ (ps)
PC	-2.1316	0.9087	$4.2725*10^9$	162	4.2
CM	-12.5797	31.6496	$1.4881*10^{11}$	4.6580	

From the table 4 we found the absolute deviation between theoretical calculations and experimental data for half-live in the state of pure configuration is $\Delta t^{PC}_{1/2}=|t^{PC}_{1/2}-t^{\exp.}_{1/2}|=157.8(\text{ps})$ for $^{78}_{34}Se$, this deviation in the states of configuration mixing became for all allowed states on the full space $1p\text{-}0f\text{-}0g_{9/2}$ is $\Delta t^{CM}_{1/2}=|t^{CM}_{1/2}-t^{\exp.}_{1/2}|=0.458(\text{ps})$.

Table 5. calculated values of transition amplitude, reduced transition probability transition probability and half-live in the pure configuration and in the configuration mixing compared with the experimental value for $^{78}_{35}Br$

CONFIGURATION	$(a_fb_f;J_f\|\|\hat{M}_1\|\|a_ib_i;J_i)$ (μ_N/c)	$B(M1:1^+\rightarrow 2^+_{gs})$ $(\mu_N/c)^2$	$T(M1)$ $(1/\text{sec})$	$t_{1/2}^{th.}$ (ns)	$t_{1/2}^{\exp.}$ (ns)
PC	-1.7815	0.6347	1.8671*109	0.3712	7.5
CM	0.3949	0.0312	9.1727*107	7.5566	

and from the table 5 we found the absolute deviation for half live in the state of pure configuration is $\Delta t^{PC}_{1/2}=7.1288(\text{ns})$ for $^{60}_{29}Cu$, this deviation in the states of configuration mixing became for all allowed states on the full space $1p\text{-}0f\text{-}0g_{9/2}$ is $\Delta t^{CM}_{1/2}=|t^{CM}_{1/2}-t^{\exp.}_{1/2}|=0.0566(\text{ns})$

Table 6. calculated values of transition amplitude, reduced transition probability and half-live in the pure configuration and in the configuration mixing for compared with the data for $^{78}_{36}Kr$

CONFIGURATION	$(\text{BCS}\|\|Q_2\|\|ab;2^+)$ (efm^2)	$B(E2:2^+\rightarrow 0^+_{gs})$ (e^2fm^4)	$T(E2)$ $(1/\text{sec})$	$t_{1/2}^{th.}$ (ps)	$t_{1/2}^{\exp.}$ (ps)
PC	2.0569	0.8461	$2.0661*10^9$	335	3.7
CM	-19.6567	77.277	$1.8870*10^{11}$	3.6734	

and from the table 6 we found the absolute deviation for half live in the state of pure configuration is $\Delta t^{PC}_{1/2}=331.3(\text{ps})$ for $^{78}_{36}Kr$, this deviation in the states of configuration mixing became for all allowed states on the full space $1p\text{-}0f\text{-}0g_{9/2}$ is $\Delta t^{CM}_{1/2}=0.0266(\text{ps})$. Also taking configuration mixing into consideration in electromagnetic transitions enhanced considerably the results more than the case of pure configuration when comparison with experimental data for this nuclei.

And we can say the configuration mixing is powerful tool to calculate transition probability for studied nuclei more than pure configuration.

9. CONCLUSIONS

Shell model calculations and the BCS theory with effective interactions MSDI between two quasiparticles were performed using the AUSM code to reproduce first exited states and the reduced transition probabilities $B(M\lambda; J_i \rightarrow J_{gs})$ for A= 78, in the pure configurations and the configuration mixing for all allowed states on the full valence space $1p\text{-}0f\text{-}0g_{9/2}$. Good agreement was obtained by comparing this calculations in configuration mixing with the recently available experimental data. The MSDI can be used as a good instrument for reproducing these calculations in the valence space $1p\text{-}0f\text{-}0g_{9/2}$ .Also, when the valance nucleons is large. Obtain better agreement with experiment data, we can add the dimensions of the core excitation admixture, and taking into account more than two quasiparticle in configuration mixing. Also, it's possible to improve the calculation by increasing the size of the model space and performing a more careful analysis of the single particle energy and the interaction parameters. This work can be extended to study more chain of isotopes to have better understanding of nuclear structure and effective interactions and the possible ways to modify them to be more agreeable with theexperimental data.

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